Log-periodic self-similarity in the S&P 500 index

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The suggestion that financial dynamics may be governed by phenomena analogous to criticality in the statistical physics sense and, especially, the related subtle concept of log-periodicity [1, 2] proves exciting and at the same time somewhat controversial [3]. In its conventional form criticality implies a scale invariance which, for a properly defined function $F(x)$ characterizing the system, means that

$$F(\lambda x) = \gamma F(x). \tag{1}$$

A positive constant γ in this equation describes how the properties of the system change when it is rescaled by the factor λ. One obvious solution to this equation is:

$$F_0(x) = x^\alpha, \tag{2}$$

where $\alpha = \ln(\gamma)/\ln(\lambda)$. It represents a standard power-law that is characteristic of continuous scale-invariance and α is the corresponding critical exponent. The zig-zag character of financial dynamics attracts attention to the general solution of Eq. (1):

$$F(x) = x^\alpha P(\ln(x)/\ln(\lambda)). \tag{3}$$

P denotes a periodic function of period one. The dominating scaling (2) thus acquires a correction that is periodic in $\ln(x)$. This solution can be interpreted in terms of discrete scale-invariance [4] and a complex critical exponent [5]. A functional form of P is not determined at this level. It only demands that if

$$x = |T - T_c|, \tag{4}$$

where T denotes the ordinary time labeling the original price time series, represents a distance to the critical point T_c, the resulting spacings between the corresponding consecutive repeatable structures at x_n (i.e., minima or maxima) of the log-periodic oscillations seen in the linear scale follow a geometric contraction according to the relation

$$\frac{x_{n+1} - x_n}{x_{n+2} - x_{n+1}} = \lambda. \tag{5}$$

The critical points coincide with the accumulation of such oscillations and, in the context of the financial dynamics, it is this effect that potentially can be used for prediction provided λ is really well defined and constant.

As an example we investigate the Standard & Poor's 500 index in the period from 1997 to 2002. Keeping in mind that there exists some freedom in choosing a specific form of the periodic function P in Eq.(3), which imposes a serious restriction on the mathematical rigour of the corresponding methodology, we take the first term of its Fourier expansion,

$$P(\ln(x)/\ln(\lambda)) = A + B\cos(\frac{\omega}{2\pi}\ln(x) + \phi). \tag{6}$$

This of course implies that $\omega = 2\pi/\ln(\lambda)$. A unit used to measure x (equivalently T) can be absorbed into ϕ. With $\lambda = 2$ we then try to obtain the best representation of the oscillatory structure seem in the real market during this period. The result of such an analysis is presented in Fig. 1. It clearly indicates that the modulus of the *cosine* in Eq. (6)

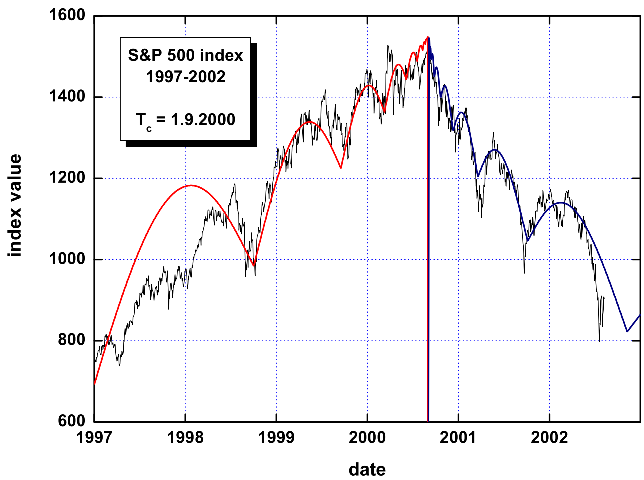

Fig. 1: S&P500 from 1997 till end of August 2002. The solid lines serve as a reference illustrating the log-periodically accelerating and decelerating scenarios with a common time of crash $T_c = 1.9.2000$. The modulus of the *cosine* is found here to constitute a better representation. The preferred scaling factor $\lambda = 2$ is used everywhere.

provides a better representation for the log-periodic modulation. Secondly, it provides independent evidence that the real date T_c marking reverse of the upward global trend is the begining of September 2000, as it is exactly at this time that the decelerating log-periodic oscillations accompanying the decline start. Such an impressive synchronization of the end of a log-periodically accelerating bubble phase with the begining of log-periodically decelerating "anti-bubble" [6] phase is spectacular, and can indeed be considered as an extra argument in favour of the consistency of the log-periodic scenario may offer. Furthermore, the same preferred scaling factor $\lambda = 2$ has been used in the analytical representation for the decelerating phase shown in Fig. 1, and it looks optimal.

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